ADDRESSING DIFFERENT COGNITIVE LEVELS FOR ON-LINE LEARNING

Nestor Aguilera
PEMA/INTEC, ARGENTINA
aguilera@arcride.edu.ar

George Fernandez & Gary Fitz-Gerald
School of Computer Science and Information Technology
Department of Mathematics and Statistics
RMIT University, AUSTRALIA
{gfernandez, garyfitz}@rmit.edu.au

Abstract

On-line teaching and learning with the help of a learning environment naturally shifts the balance from a teacher-centred to a student-centred approach. When teachers and students are not often in face-to-face contact, the learning has to be carefully planned and adequately supported by the teaching environment to induce learning at the appropriate level. In previous work, we have argued that in on-line learning the main focus should be student activities ("what students do"), rather than what teachers do (Fernandez, 2001; Fernandez, John & Netherwood 2001; John, Netherwood, Sudarmo & Fernandez, 2002). We describe here an approach to teaching Mathematics by targeting clearly identified and expressly stated learning outcomes at different appropriate learning levels. Our approach relies on the use of WebLearn, a student-centred learning environment that possesses symbolic manipulation capabilities via the use of a Maple engine.

In this paper we describe how WebLearn can be used to provide automated formative and summative assessment of university students studying Mathematics. The discussion centres on how a subject may be broken down using learning objectives at different levels of abstraction, and how these may be addressed by a lecture plan closely aligned with the support provided by the use of a student-centred learning environment.

Keywords

WebLearn, Bloom's Taxonomy, On-line Learning, Student-Centred Learning

Introduction

In on-line teaching and learning there is much less teacher-student face-to-face contact than traditionally. This implies a need to change the emphasis from a teacher-centred to a student-centred approach. The main purpose of teaching ceases to be to transmit information in a clear and organised manner, to manage the instruction process properly (see Level 1 and Level 2 in Prosser & Trigwell, 1998). Consequently, the process is mostly concerned with what students have to do to learn, and with enhancing as much as possible any staff-student interaction. Although learning environments with automated feedback have been used successfully, this has been mostly for rote learning by focusing on drilling exercises. At the tertiary level, however, the challenge is to encourage students to develop and use their higher cognitive processes when learning. We argue that in on-line teaching and learning this may be possible by engaging students in appropriate self-directed learning activities that foster question, reflection and analysis, rather than mere repetition. In on-line teaching, teachers act mostly as learning mediators (Laurillard, 1993). Thus, with the support of an appropriate environment, well-structured learning tasks should induce consideration, inquiry and discovery in the students, and thereby support their learning process at the higher levels required for tertiary studies.

By referring to higher cognitive processes, we are implying the existence of a hierarchical classification of learning. Bloom's “Taxonomy of Learning Objectives” (Bloom, 1964), for example, clearly distinguishes between different levels of learning. The taxonomy identifies six levels within the cognitive domain, from the simple recall or recognition of facts (Knowledge) at the lowest level, through to increasingly more complex and abstract mental levels (Comprehension, Application, Analysis, Synthesis) up to the highest order (Evaluation). The taxonomy attempts to establish not only what learning topic or concept is under consideration, but also the required level of student learning. To induce learning at the higher cognitive levels it is necessary to design activities that appropriately address them. The developmental strategy should focus on providing an environment that requires students to use their capacity to abstract, analyse, discover, formulate and hypothesise, rather than merely remembering factual information. (See Chalmers & Fuller, 1995)

As we mention above, learning environments with automated responses have been mainly used for drilling activities that address Knowledge and Comprehension, the lower levels of the taxonomy. The main difficulty associated with addressing the higher cognitive objectives is that activities that encourage students to extrapolate, propose, abstract and formulate are highly dependent on what students already know or have seen in their previous experience. An activity requiring students to propose a new use for a given technique, for example, will be rendered inoperative if students have seen it used in the new way before. Thus, the use of this type of learning environment for higher-level learning should be accompanied by an appropriate teaching plan that prepares the students to carry out the activities, but without `giving away the plot'. The information (lecture, on-line material) should be designed and presented in such a way as to make it possible for students to make the higher level cognitive contributions. Carefully aligned, lecture and practical work material will leave open questions for the students to fill in the gaps by exploration and extrapolation.

However, few environments are capable of adequately supporting learning processes at the higher levels of the taxonomy, since they are restricted to questions with a given simple format, such as Multiple Choice, Multiple Answer, or Short (Key) Text. In Mathematics, the capacity for handling computer-based symbolic manipulation is typically afforded by specialised packages such as Maple or Mathematica. These systems provide an environment with which students can interact in mathematical terms, since they include specialised ‘engines’ that interpret abstract mathematical language. On the other hand, environments such as WebLearn have been designed to present questions to students and analyse their answers against predefined correct answers supplied by the lecturers.

Our approach combines the generation, presentation and feedback capabilities of WebLearn with the analysis capabilities of the Maple engine. When required, WebLearn can generate a quiz (formative) or a test (summative) according to the lecturer's instructions. Students' answers are captured by WebLearn and fed through Maple. The response from Maple is then analysed by WebLearn, massaged into an appropriate format, and fed back to the students. This makes possible the correct evaluation of questions with no unique answer, for example, thereby providing a proper assessment of *any* correct answer provided by the students.

The next sections present an integrated teaching plan and the use of the scheme to achieve learning at all levels of the taxonomy.

An Illustrative Integrated Teaching Plan

In this paper, an integrated example illustrating the different levels for the teaching and testing of linear equations in three-dimensions is considered. A structure is provided involving three lectures that develop the chosen topic material and, following each lecture, a related set of questions that explores the various levels of learning achieved. This lecture material may be delivered face-to-face, completely on-line for distance learning, or a mix of both modes. If completely on-line, the proposed exercises may be developed, presented and delivered exclusively in the WebLearn environment.

First Lecture

In this lecture a linear relationship is introduced, supported by examples and illustrative drawings, and appropriate (face-to-face or on-line) lecture material. The three standard forms of a linear relationship are provided:

$$ax + by + cz = 0, \quad ax + by + cz = k \quad \text{and} \quad z = mx + ny$$

where a, b, c and k can take positive, negative or zero values. Transformations from any one of these forms into each of the others is demonstrated.

Practical Work
This may be presented on-line, using WebLearn to pose questions to the students and provide feedback in the way suggested in Section 3.

Recall (lowest level):
Determine if a given triplet of values (x, y, z) satisfies the relationship. Ask whether given triplets satisfy the relationship.

Solve, Interpret (medium level):
Given x, calculate any y and z such that the triplet satisfies the relationship. Given x, calculate all y and z such that the triplet satisfies the relationship. Provide a general interpretation of a triplet (x, y, z) that satisfies the relationship.

Formulate (higher level):
Solve linear equation problems with a word statement.

Extrapolate, Discover (higher level):
Draw a set of points in R3 that satisfy a linear relationship (a few, quite a few). Extrapolate that all points satisfying a linear relationship belong to a plane. Discover the meaning of the constant k above, both zero and non-zero. Find out what straight lines are included in the plane.

Second Lecture

Demonstrate (prove) that all points satisfying a linear relationship are on a plane. Describe different forms, including parametric, of equations of lines and planes.

Practical Work
This may be presented on-line, using WebLearn to pose questions to the students and provide feedback in the way suggested in Section 3.

Interpret (medium level):
Write two linear equations. What does it mean that a point (x, y, z) satifies both?

Hypothesise (higher level):
Determine in general what it means that a point satisfies two linear equations. Determine what it means that a set of points satisfies two linear equations. Solve a 3x3 system by finding x, y and z. Show graphically all possible situations. Formulate a general statement expressing all the solutions of a 3x3 system of equations.

Third Lecture
Demonstrate the general form of the solution of a 3x3 system. Give a general interpretation of the normal to a plane and how it may be obtained using the cross product.

Practical Work
This may be presented on-line, using WebLearn to pose questions to the students and provide feedback in the way suggested in Section 3.

Interpret (medium level):
Given the equation of a plane, find another plane parallel to it. Find more than one.

Systematise (higher level):
Determine if two planes are parallel. Determine if two parallel planes are actually the same. Interpret with respect to solutions of a system of equations. Establish general conditions for two planes to be parallel. Determine parameters so one plane is parallel to another. Determine parameters so both are the same.

Although our focus here is the teaching of three-dimensional aspects of linear equations, it will be clear how a similar process can be employed to other topics in Mathematics, and to other disciplines.

Sample Questions

In this section, a variety of questions are discussed to ascertain the level of understanding a student has of the topic and how each may be implemented using WebLearn. The discussion includes the ‘cognitive path' that students would need to follow, from the information/knowledge provided by the lecture material to the one required to supply a correct answer.

In all the sample questions appearing in this paper, specific values that appear as parameters in the question (such as coordinates of points and the components of normal vectors to planes etc) are random values generated using WebLearn. Irrespective of the particular values that are presented to a student at any one attempt at such a question, the marking procedure—that invokes the symbolic manipulation package Maple—tests for correctness or otherwise of the answer specific to the values appearing in the rendered question. Moreover, if the answer specified is incorrect, commonly occurring misconceptions are checked for using the intelligence of Maple and appropriate feedback provided to the student via the WebLearn interface. Mathematical expressions are currently rendered using the WebEQ applet, but once MathML is supported by the commonly used browsers this will be used instead; recent releases of Maple are equipped to provide output in this format.

Lowest Level
Suitable WebLearn questions at this level are:

1. Determine if the set of values `x = 1`, `y = -2` and `z = 6` satisfy the linear equation

```
3x - 4y + 6z = 21
```

Within WebLearn the answer to such a question will be requested using a fill out form:

```
The triple (1, -2, 6) satisfies the equation

            3x - 4y + 6z = 21 [          ]

(Enter either True or False in the box provided)
```

The correct response for the parameters specified above would be

```
The triple (1, -2, 6) satisfies the equation
```

`3x - 4y + 6z = 21` [False]

`(Enter either True or False in the box provided)`

2. Determine a parametric equation of the line passing through the points `(3, -5, 7)` and `(-2, -4, 11)`. Your answer should be expressed in terms of an arbitrary parameter t.

Within WebLearn the answer to such a question will be requested using a fill out form.

x = [ ]
y = [ ]
z = [ ]

One possible correct response for the parameters specified above would then be

x = [5*t - 2]
y = [-t -4]
z = [-4*t + 11]

To successfully answer this question, the student needs to determine the direction of the line from the information given and to recognise that either of the given points can be used as part of their answer. The marking scheme checks that a correct direction has been determined, and if not provides a correct version with an explanation of how this direction can be obtained using the particular values presented to the student. It also checks if only one of the supplied components is incorrect and supplies the correct value as informative feedback.

Medium Level

Suitable WebLearn questions at this level are:

1. For `x = -4`, determine any pair of values of y and z that satisfy the linear equation

`5x + 7y - 2z = -11`

Within WebLearn the answer to such a question will be requested using a fill out form:

`Suitable values of y and z are:`

y = [ ] and z = [ ]

A correct response for the parameters specified above is

y = [1] and z = [-1]

2. For `x = -4`, determine all pairs of values of y and z that satisfy the linear equation

`5x + 7y - 2z = -11`

The answer would be requested to be expressed in terms of an arbitrary parameter t.
Within WebLearn the answer to such a question will be requested using a fill out form:

`All values of y and z are:`

y = [ ] and z = [ ]

A correct response for the parameters specified above is

y = [9/7 + 2/7*t] and z = [t]

3. Determine the equation of a plane parallel to

-3x + 11y + 17z = -21.

Within WebLearn the answer to such a question will be requested using a fill out form:

`The equation of a parallel plane is`

[ ]

A correct response for the plane specified above is:

`The equation of a parallel plane is`

[3*x – 11*y – 17*z = 90]

Determine the equation of a plane that contains the points (2, 0, 3) and (0, -12, 0), Within WebLearn the answer to such a question will be requested using a fill out form:

`The equation of such a plane is`

[ ]

A correct response for the points specified above is:

`The equation of such a plane is`

[3*x - y + 2*z = 12]

Highest Level

Suitable WebLearn questions at this level are:

1. Determine a Cartesian equation of the plane containing the lines

L_1: (x, y, z) = (1, 0, -1) + t(2, -4, 5)

and

L_2: (x, y, z) = (-2, 3, 0) + s(7, -11, 9)

In answering such a question the student will be presented with a fill out form:

`A Cartesian equation of the plane is`

[ ]

A correct response for the parameters specified above would then be

`A Cartesian equation of the plane is`

`19*x + 17*y + 6*z = 13`

To successfully answer this question, the student:

- either recognises that since each line lies in the plane, their directions are both perpendicular to the plane's normal. Hence the normal is parallel to the cross product of these directions (2, -4, 5) and (7, -11, 9).

- or needs to determine three non-collinear points on the plane. Suitable points would be `(1,0, -1)` (`t = 0` in the equation for line L_1), `(-2, 3, 0)` (`s = 0` in the equation for line L_2) and `(3, -4, 4)` (`t = 1` in the equation for line L_1) and then find the plane's normal using two directed line segments joining one of these points to each of the others.

2. Determine the distance of the point `P(-2, 1, -4)` from the line

`L: (x, y, z) = (2, 2, -7) + t(-1, 2, -5).`

In answering such a question the student will be presented with a fill out form:
`The distance of the point P(-2, 1, -4) from the line L is approximately`

[ ] `units`

The correct response for the parameters specified above would then be

`The distance of the point P(-2, 1, -4) from the line L is approximately`

`4.443` `units`

To successfully answer this question, the student:

- either needs to determine a point Q on the line—such as `(2, 2, -7)`, and then recognise that the distance between the point Q and the point R (representing the closest point on the line to P) is the scalar projection of the directed line segment joining the points P and Q onto the vector in the direction of the line. Then, since the distance between the points P and Q is known, the distance between the points P and R can be obtained using Pythagoras' theorem applied to the triangle PQR

- or determine the coordinates of the point R lying on the line L such that the directed line segment joining the points P and R is perpendicular to the line; and then calculating the distance between this point R and the given point P.

References

Bloom B.S. (1964). *Taxonomy of Educational Objectives*, David McKay Company Inc, New York

Chalmers D. & Fuller A. (1995). *Teaching for Learning at University: Theory and Practice.* Edith Cowan University, Perth.

John S., Netherwood G., Sudarmo & Fernandez G. (2002). Learning Taxonomy Analyses Of Student-Based Activities Using The Lego Mindstorms System. *Proceedings of the 13th AAEE Conference*. Canberra, ACT, Australia (To appear).

Fernandez G. (2001). WebLearn: A CGI-Based Environment for Interactive Learning. *Journal of Interactive Learning Research*. Vol 12, No 2, pp 265-280.

Fernandez G., John S. & Netherwood G. (2001). Objective-Based Teaching of Science and Engineering With an On-line Student-Centred Environment. *Proceedings of the 12th AAEE Conference.* QUT, Brisbane, Australia, pp 332-337.

Laurillard D. (1993). *Rethinking University Teaching: A Framework for the Effective Use of Educational Technology*. Routledge. London.

Prosser M. & Trigwell K. (1998). Teaching for Learning in Higher Education, Open University Press. Buckingham.